d. Example 4

Obtain an expression for the entropy change in an RK gas when the gas is isothermally compressed. Determine the entropy change when superheated R–12 is isothermally compressed at 60ºC from 0.0194 $m^3\ kg^{-1}$ (state 1) to 0.0126 $m^3\ kg^{-1}$ (state 2). Compare the result with the tabulated value of $s_1 = 0.7259$, $s_2 = 0.6881$.

Solution

Consider the RK state equation

$$P = RT/(v-b) - \mathbf{a}/(T^{1/2} v(v+b)) \qquad (A)$$

Note that the attractive force constant **a** is different from "a" Helmholtz function. From the third relation Eq. (22) and Eq. (A),

$$(\partial s/\partial v)_T = (\partial P/\partial T)_v = R/(v-b) + (1/2)\, a/(T^{3/2} v(v+b)). \qquad (B)$$

Integrating Eq. (B),

$$s_2(T,v_2) - s_1(T,v_1) =$$

$$R\ln((v_2-b)/(v_1-b)) + (1/2)(a/(T^{3/2}b)) \ln(v_2(v_1+b)/(v_1(v_2+b))). \qquad (C)$$

From Table 1 for R–12, T_c = 385 K, and P_c = 41.2 bar. Therefore $\bar{a}$ =208.59 bar $(m^3\ kmole^{-1})^2\ K^{1/2}$, and $\bar{b}$ = 0.06731 $m^3\ kmole^{-1}$. The molecular weight M = 120.92 kg $kmole^{-1}$, and

$a = \bar{a}/M^2 = 208.59$ bar $(m^3\ kmole^{-1})^2 K^{1/2} \div 120.92^2 (kg\ kmole^{-1})^2$

$= 1.427$ k Pa $(m^3\ kg^{-1})^2\ K^{1/2}$, and

$b = \bar{b}/M = 0.557\times10^{-3}\ m^3\ kg^{-1}$.

Since, $R = 8.314 \div 120.92 = 0.06876\ kJ\ kg^{-1}\ K^{-1}$,

$s_2 - s_1 = 0.06876 \ln[(0.0126 - 0.000557) \div (0.0194 - 0.000557)]$

$+ (1/2)\{172.5 \div (333^{1.5}\ 0.000557)\} \ln [0.0126 (0.0194 + 0.000557)$

$\div \{0.0194 \times (0.0126+ 0.000557)\}]$.

$= - 0.06876 \times 0.448 - 0.211 \times 0.01495 = -0.03396\ kJ\ kg^{-1}\ K^{-1}$.

Solution

Since the process is adiabatic and reversible we will use the relation $(\partial T/\partial P)_s = Tv\beta_P/c_P$ or $(\Delta T/\Delta P)_s = T_o v\beta_P/c_P.=$ $\{250K \times 1.1\times10^{-4} m^3 kg^{-1} \times 48x10^{-6} K^{-1}\} /0.372 kJkg^{-1}K^{-1} = 3.548x10^{-6}$ K/Kpa and $\Delta P = 100 \times 1000$ Kpa. Hence $dT_s = 0.36$ K and T will rise to 250.36 K.

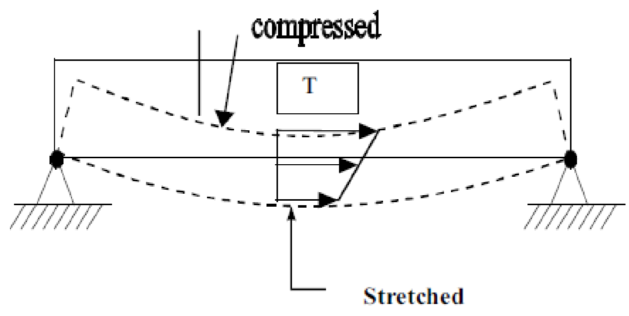

Figure 6: The bending of a beam.

Applying the First law to an adiabatic reversible process
$du_s = -Pdv_s$.
Recall from Eq. (37) that
$dv_s = -dT_s\ \beta_T c_v/(T\ \beta_P)$, i.e.,
$du_s = \{P\ \beta_T c_v/(T\ \beta_P)\} dT_s$

Integrating and assuming that P is not a function of temperature and remaining at an average value of ***500*** bar.

$du_s = \{500 \text{ bar} \times 7.62\times10^{-7} \text{ bar}^{-1} \times 0.364 \text{ kJ kg}^{-1} \text{ K}^{-1}$
$\div (250 \text{ K} \times 48\times10^{-6} \text{ K}^{-1})\}\ \ 0.36 \text{ K} = 0.00416 \text{ kJ kg}^{-1}$.

The temperature after the load is removed is 250 K, since the process is reversible.